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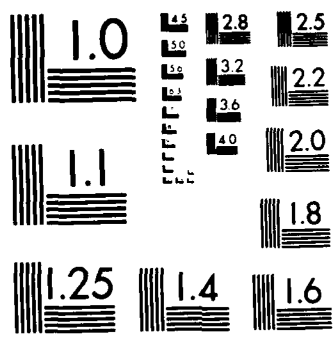
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The funds from AFOSR-84-0256 were used to purchase a VAX 11/750. A total of 33 reseach papers by 7 U. Arizona Mathematicians were written with extensive use being made of the VAX. On addition 2 Ph.D. students wrote doctoral dissertations which were heavily oriented toward scientific computation.

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This is an excellent record of achievment and a clear indication of the benefit the AFOSR equipment can produce.

This report is certainly acceptable.

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Christopher Jones

"On the infinitely many solutions of a semilinear elliptic equation", (with T. Kuepper), accepted by SIAM Jrnl. of Mathematical Analysis.

"Global dynamical behavior of the optical field in a ring cavity", (with S. Hammel and J. Moloney), accepted by the Jrnl. of the Optical Society of America, Series B.

"Chaos and coherent structures in partial differential equations", (with A. Aceves, H. Adachihara, J.C. Lerman, D. McLaughlin, J. Moloney and A.C. Newell), Physica 18D, 85-112 (1986).

"Stabilty of nonlinear waveguide modes", (with J. Moloney), submitted to Phys. Rev. Letters.

"Invariant manifold theorems with applications", to appear in Proc. NATO Conference on Nonlinear Analysis, April, 1985, Italy.

"Invariant manifolds for semilinear partial differential equations", (with P. Bates), submitted to SIAM Review.

David McLaughlin

"A new class of instabilities in passive optical cavities", (with J. Moloney and A.C. Newell), Phys. Rev. Lett. 54, 681 (1985).

"Chaos and Coherent Structures in Partial Differential Equations", (with A. Aceves et al.) Physica 18D, 85-112 (1986).

Jerome V. Moloney

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"Chaos and turbulence: is there a connection?", to appear in the Special Proc. of Conference on Mathematics Applied to Fluid Mechanics and Stability dedicated in memory of Richard C. DiPrima, to be published by SIAM, 1986.

"The shape of stationary dislocations", (with D. Meiron), Physics Letters, Vol. 113A, No. 5, (1985).

"Chaos and coherent structures in partial differential equations", (with A. Aceves et al.) Physica 18D, 85-112 (1986).

"Two-dimensional spatial patterns in ring cavities", (with D. McLaughlin and J. Moloney), to be submitted.

"The Hirota Conditions", with (Y. Zeng), submitted to J. Math. Physics.

"Benjamin-Feir turbulence in binary mixture convection", (with H. Brand and P. Lomdahl), to appear in Physica D.

Gregory Baker

"Boundary Integral Methods for Axisymmetric and Three-Dimensional Rayleigh-Taylor Instability Problems", (with D. Meiron and S.A. Orszag), Physica 12D, 19 (1984).

"Boundary Integral Techniques for Multi-connected Domains", (with M.J. Shelley), to appear in J. Comp. Physics (1986).

"Rayleigh-Taylor Instability of Fluid Layers", (with R.L. McCrory, C.P. Verdon and S.A. Orszag), to appear in J. Fluid Mech. (1986).

Nicholas Ercolani

"The Geometry of Baker Functions and the Neumann Problem", (with H. Flaschka), to appear in the Proc. of the Royal Society of London (1984).

"The Geometry of the Modulational Instability", (with G. Forest and D. McLaughlin), 1984.

"Monopoles and Baker Functions", (with A. Sinha), 1984.

Daniel Meiron

"Generalized Methods for Free Surface Flow Problems II: Radiating Waves", (with G.R. Baker and S.A. Orszag), in press.

"Boundary Integral Methods for Axisymmetric and Three-Dimensional Rayleigh-Taylor Instability Problems", (with G.R. Baker and S.A. Orszag), Physica 12D, 19-31 (1984).

"The Shape of Stationary Dislocations", (with A.C. Newell), Physics Letters 113a, 289 (1985).

"Difficulties with Three-Dimensional Weak Solutions for Inviscid Incompressible Flow", (with P.G. Saffman), sub judice, Physics of Fluids, (1986).

"Selection of Steady States in the Two-Dimensional Symmetric Model of Dendritic Growth", to appear, Phys. Rev. A, 33, (1986).

Michael Shelley

Ph.D. thesis: "The Application of Boundary Integral Techniques in Multiply Connected Domains"

"Order Preserving Approximations to Successive Derivatives of Periodic Functions by Iterated Splines", (with G. Baker), submitted for publication to SIAM Jrnl. of Numerical Analysis (1986).

Hatsuo Adachihara

"Chaos and Coherent Structures in Partial Differential Equations", (with A. Aceves et al.), Physica 18D, 85-112 (1986).

Alejandro Aceves

Mr. Aceves is a Ph.D. candidate in the Applied Mathematics Program working on two problems: trapping of nonlinear pulses on nonlinear waveguides by slits which have an amplitude independent refractive index; curious behavior which arises when delay effects are included in the Ikeda map, this map arises in the study of optically bi-stable cavities.

Nonlinear Optics:

a) We are numerically integrating the differential-delay system that describes the propagation of light in a ring cavity.

The purpose of this research is to identify the bifurcation sequence in the [illegible] parameter space. [illegible] preliminary results show that this is not the case, and our aim is to understand the reason for this.

b) We are studying the propagation of solitons on waveguides. Some analysis has been done, describing how the soliton behaves assuming no radiation is being created.

From this analysis, one has to solve a system of ODE's for the parameters of the soliton.

This has been done on the VAX. We have also integrated numerically the full problem to see the effects of radiation.

Ali Ouarzeddini

His work has been concentrated on solving numerically the Ginzburg-Landau equation, sometimes referred to as the Newell-Whitehead equation, which has many physical applicatons. The equation is:

$$W_t - (\gamma_r + \gamma_i)W_{xx} = \chi W - (\beta r + \beta i)W^2 W^*$$

The boundary conditions are periodic.

Different parameter values are used to check the stability of the x-independent and the soliton solutions. The equation is integrated in time using a pseudo-spectral method. The linear part of the equation is solved for half time step using the FFT (the Fast Fourier Transform). The nonlinear part is solved exactly for the remaining half-time step. Data is collected and analyzed.

Christopher Jones

Current Research

There are two major projects that form a focus for my research at the moment. The first is a stability problem for travelling waves in a fast-slow system of reaction-diffusion equations. The travelling wave is constructed by piecing together solutions of reduced problems and the question is to understand how the stability information for these reduced problems is put together to determine the stability of the full problem. This is related to my earlier work on the FitzHugh-Nagumo equations but significantly more difficult as the underlying phase space dimension is 4 rather than 3. R. Gardner of the University of Massachusetts proved the existence of the waves and the extra space dimension translates into non-trivial behavior in the slow part of the system. The understanding of the effect of this new feature on the stability problem is quite subtle. Gardner and I are working on this problem jointly.

The second problem relates to standing wave solutions of the nonlinear Schrödinger equations. I have proved an instability result and applied it to nonlinear optical waveguides with Moloney. I am currently involved in extending the results to a wider class of waves. But what is more interesting is that the results have suggested some striking connections between the methods of proof used to find the waves in the first place, namely, the dynamical systems approach (as developed by Küpper and myself) and the variational approach (as develcped by Strauss and Berestycki-Lions). These techniques have up until now developed independently and a relationship between them will, I believe, open up the possibility of exciting new results.

Instability of Standing Waves for

Nonlinear Schrödinger Type Equations

by

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This work is dedicated to the memory of Charles Conley

This research was supported in part by the National Science Foundation NSF Grants DMS 8501961 and DMS 8507056, Air Force Office of Scientific Research AFOSR Grant #83-0027, and Army Research *Office* ARO Contract DAAG-29-83-K-0029.

Instability of Standing Waves in Nonlinear Optical Waveguides

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PACS numbers: 02.78 42.20

Abstract

A new mathematical instability technique is presented and applied to determine the stability properties of a physically important class of standing waves in nonlinear planar optical waveguides. The method is illustrated by a case where soliton perturbation techniques or variational methods are inapplicable.

On the Infinitely Many Solutions of a Semilinear Elliptic Equation

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Lefschetz Center for Dynamical Systems

New Class of Instabilities in Passive Optical Cavities

D. W. McLaughlin, J. V. Moloney, and A. C. Newell
Applied Mathematics Program and Optical Science Center, University of Arizona, Tucson, Arizona 85721
(Received 2 July 1984)

In this Letter we show that the fixed points of the Ikeda map are more unstable to perturbations with a short-scale transverse structure than to plane-wave perturbations. We correctly predict the most unstable wavelength, the critical intensity, and the growth rates of these disturbances. Our result establishes that, for a large class of nonlinear waves, spatial structure is inevitable and drastically alters the route to chaos. In an optical cavity the consequence is that the period-doubling cascade is an unlikely scenario for transition to optical chaos.

PACS numbers: 42.65.−k

In this Letter, we announce a new and unexpected result, an instability whose consequences have ramifications for a large class of nonlinear wave problems whose dynamics can be described by envelope equations. Specifically, it deals with periodically forced field equations of the universal nonlinear Schrödinger type. This instability changes the whole character of the route of the system from a simple to a turbulent state. It generates spatial structure, and the subsequent onset of chaotic behavior completely bypasses the period-doubling scenario which is relevant if spatial structure is ignored. Moreover, the scenario which does emerge has a universal character of its own. Examples of this phenomena are found in optics, either in the transmission along optical fibers or in optically bistable cavities.[1] It is in the latter context that this work is presented.

In this problem we are interested in the long-time state of a continuous laser signal which is recirculated through a nonlinear medium. In examining one particular manifestation of optical bistability (a ring cavity with Kerr nonlinearity), Ikeda[2] wrote a map expressing the (complex) amplitude g_{n+1} of the electric field E on the $(n+1)$st pass through the cavity as a function of electric field amplitude on the nth pass;

$$g_{n+1} = a + Rg_n \exp[i\phi_0 + ipLN(I)/2]. \tag{1}$$

In (1), a is the amplitude of the input field, $R < 1$ the reflectivity of the mirrors, ϕ_0 the detuning parameter, p is (effectively) the length of the nonlinear medium, and $N(g_n g_n^*)$ measures its nonlinear response. Two cases are usually studied: (1) the saturable medium, $N(I) = -(1+2I)^{-1}$, (2) the Kerr medium, $N(I) = -1+2I$, which is the small intensity limit of the saturable case. Equation (1), called the Ikeda map, is a two-dimensional invertible map and exhibits a variety of behavior which is already well documented in the literature.[2-4] In various parameter ranges (the two parameters which are varied are a and p), one finds multiple fixed points (see Fig. 1) and sequences of period-doubling bifurcations leading to chaotic attractors.

The map (1) invokes the plane-wave approximation in which diffraction effects are neglected. The purpose of this Letter is to point out that this assumption is not justified even for cases in which the input beam is very slowly varying in the transverse direction $\mathbf{x}(x,y)$ and the Fresnel number is large. The reason is that the fixed point solutions of the plane-wave map (1) are more unstable to perturbations with a short-scale transverse structure than they are to perturbations with plane-wave structure. To emphasize this point, the numerical experiment discussed in this Letter is run at a parameter value p for which the fixed points of the Ikeda map are stable!

This discovery has important ramifications. First, it shows that the initial bifurcation of the system introduces an extra dimension into the problem, a short-wave transverse excitation of temporal period two. This extra dimension affects significantly the subsequent behavior of the system. As the stress parameters are raised, no period-doubling cascades into chaos

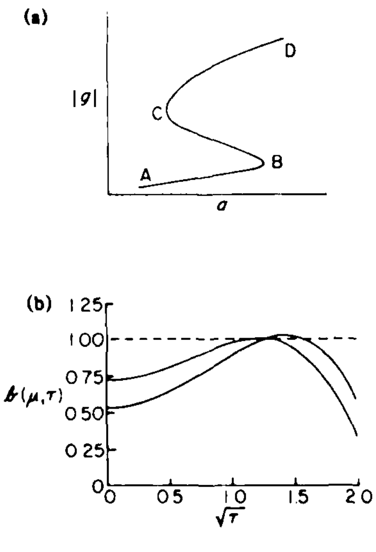

FIG. 1. (a) The multivalued response of the amplitude of the fixed point $|g|$ vs a at fixed values of p for the Ikeda map. (b) A graph of $b(\mu,\tau)$ vs $\sqrt{\tau} = \sqrt{\gamma}K$ for $\mu = pI = pgg^*$ equal to 0.11 and 0.24.

Stability of nonlinear stationary waves guided by a thin film bounded by nonlinear media

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(Received 2 December 1985; accepted for publication 3 February 1986)

The stability of stationary, TE_0-type, nonlinear, thin-film guided waves was investigated numerically for both symmetric and asymmetric planar waveguides with nonlinear cladding and substrate layers. It is found that large regions of the dispersion curves are unstable at high powers.

Unique properties[1-7] have been predicted for waves guided by thin films when one or more of the guiding media exhibit a field-dependent refractive index. Self-focusing bounding media lead to multiple new branches with power thresholds, as well as field distributions whose maxima shift from the film to the bounding media with increasing power.[1-7] In fact, this geometry has been identified as an excellent candidate for all-optical switching, with our without bistability.[5-7] To date, however, theoretical analysis has been based solely on steady-state solutions to a nonlinear wave equation which contains an intensity-dependent refractive index. The salient question is whether these wave solutions are stable on propagation, and the consequences of possible unstable regions to proposed devices. For the related problem of self-focusing of plane waves in infinite media, Kolokolov[8] has shown that the solutions are stable for $dP/d\beta > 0$ where P is the power and β is the power-dependent refractive index. Numerical propagation studies of nonlinear waves guided by the interface between a self-focusing and a power-independent medium by Akhmediev and co-workers[9,10] have led to a similar conclusion. Recently, a theory based on phase portraits has been developed[11] for the stability of TE_0 waves guided by a thin film bounded by self-focusing media, and the important conclusion of that work is that the waves are unstable on negatively sloped branches ($dP/d\beta < 0$) of the nonlinear dispersion curve, and that they are stable on positively sloped regions provided that self-focusing occurs in only one nonlinear medium. In this letter we report a test of this conclusion via a numerical investigation of the stability of TE_0 solutions for films bounded by self-focusing media.

The geometry analyzed consists of a film ($|x| < d$, refractive index n_0) bounded by two nonlinear media with low power indices n_1 and n_2, as shown in Fig. 1. Since the numerical analysis is performed in the slowly varying phase and amplitude approximation, we write the optical field as

$$E(r) = \tfrac{1}{2} W(x,z) e^{i(\beta k_0 z - \omega t)} + \text{c.c.}, \tag{1}$$

where β is the effective guided wave index, $k_0 = \omega/c$ and the variation in the amplitude term $W(x,z)$ along the propagation direction z is assumed to be small over one wavelength. The refractive index in the various media is given by

$$n^2[|W(x,z)|^2] = n_i^2 + \alpha_i |W(x,z)|^2 \quad i = 1,2 \quad (|x| \geqslant d) \tag{2a}$$

and

$$n^2[|W(x,z)|^2] = n_0^2 \quad (|x| \leqslant d)\,. \tag{2b}$$

Substituting into the nonlinear wave equation and retaining only the first derivative of $W(x,z)$ with respect to z leads to

$$2i\beta k_0 \frac{\partial W(x,z)}{\partial z} + \frac{\partial^2 W(x,z)}{\partial x^2} - k_0^2 \times \{\beta^2 - n^2[|W(x,z)|^2]\} W(x,z) = 0\,. \tag{3}$$

Analytical stability analysis of Eq. (3) is complicated by the fact that it is a partial differential equation and, furthermore, is a Hamiltonian system. The usual stability analysis for dissipative dynamical systems does not apply (unless one deliberately introduces losses into the problem). One reason for the difficulty in studying the stability of Eq. (3) is that many of the eigenvalues of the linearization of (3) lie on the imaginary axis which is precisely the condition for instability in a dissipative dynamical system. Although it has been possible[11] to perform a stability analysis for TE_m with $m = 0$, it is necessary to proceed numerically for $m \neq 0$. First, the steady-state solutions with $W(x,z) \rightarrow W(x)$ were obtained in the usual way[1-7] to obtain a field solution corresponding to a particular point on one of the nonlinear guided wave solution branches. This distribution was then assumed to be launched at $z = 0$ and Eq. (3) was solved numerically for

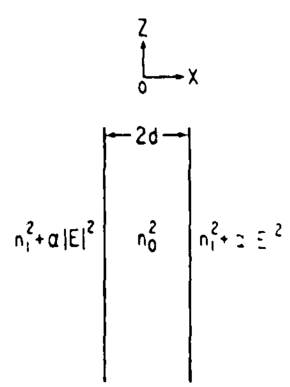

FIG. 1. Nonlinear waveguide geometry studied for wave stability.

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Chaos and Turbulence; is there a connection?

by

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Abstract. In this essay we discuss the relation of chaos, which is the unpredictable behavior associated with finite systems of ordinary differential and difference equations, and turbulence, which is the unpredictable behavior of solutions of infinite dimensional, nonlinear partial differential equations. The evidence that there is some connection, at least in certain regimes of parameter space, is sufficiently convincing to provide the motivation to search for analytical means for reducing the governing partial differential equations to either a finite system of ordinary differential equations or a much simpler partial differential equation of universal type. A successful reduction scheme must capture the spatial structure of the dominant modes accurately and we suggest ways of finding these structures in certain limiting situations. Five such schemes are presented and, in each case, the approximation is related in some way to the presence of a small parameter, near critical, nearly integrable or nearly periodic. One of these reductions leads to the complex Ginzburg-Landau equation, which has universal character, and its importance is stressed. In connection with this equation, we introduce the terms "wimpy" turbulence and "macho" turbulence to connote the crucial differences between the behavior of its solution in one and two space dimensions, a difference which has much in common with the contrast between two and three dimensional high Reynolds number flows because of vorticity production. In the final section, several ideas concerning the nature of high Reynolds number, fully developed turbulence are presented and the possible roles of singular solutions and "fuzzy" attractors are discussed. Throughout the essay, we argue that before much new progress is made, one has to understand the onset of spatial chaos, that is, the transition from a spatially regular state (possibly with a chaotic temporal behavior) to one in which the spatial power spectrum is broadband. This question is a major focus of our present research program.

This contribution is dedicated to the memory of Dick DiPrima, a good friend and long time colleague who left us too soon.

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MRC Technical Summary Report #2830

BOUNDARY INTEGRAL TECHNIQUES FOR MULTI-CONNECTED DOMAINS

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The geometry of the Hill equation and of the Neumann system

BY N. M. ERCOLANI† AND H. FLASCHKA

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Let there be given a finite-gap operator $L = d^2/dx^2 + q$ and its Baker function $\psi(x, p)$, which is analytic for p on a certain hyperelliptic curve C. It is shown that a sequence of Bäcklund transformations maps C to a projective space. This embedding can be interpreted as a matrix representation of the Hill equation by the Neumann system of constrained harmonic oscillators. The image curve, C', lies on a rational ruled surface; the structure of this surface is explained by use of ideas due to Burchnall & Chaundy (*Proc. R. Soc. Lond.* A **118**, 557–583 (1928)). Baker functions and Bäcklund transformations are then used to define a (many-to-many) correspondence between effective divisors on the curve C and points lying on a quadric, or in the intersection of two or more quadrics. This relates the theory of the Hill equation to earlier work of Knörrer, Moser and Reid. It is then shown that the Kummer image of the Jacobian of C can be realized as a hypersurface in the space of momentum variables of the Neumann system. Further projects, such as extensions to non-hyperelliptic curves, are outlined.

1. INTRODUCTION

A differential operator $L = D^2 + q(x)$, $D = d/dx$, is said to be 'finite-gap' if it commutes with a differential operator B of odd order, $[L, B] = 0$. A 'finite-gap potential' $q(x)$ is therefore a *time-independent*, or *stationary*, solution of an equation $\partial L/\partial t = [B, L]$ in the Korteweg–de Vries hierarchy. Because L and B commute, they have a common eigenfunction

$$L\psi = E\psi,$$
$$B\psi = R\psi.$$

The eigenvalues E, R are known to be related by an algebraic equation

$$R^2 = \prod_{j=1}^{2j+1} (E - \epsilon_j), \tag{1}$$

and the common eigenfunction ψ (the 'Baker function') is an analytic function on the Riemann surface (1) or, equivalently, a holomorphic section of a certain line bundle on (1). Until now, the theory of finite-gap operators has drawn mostly on the analytical aspects of Riemann surfaces and on their abstract, intrinsic geometry.

Our aim in this paper is to explain some of the *extrinsic* properties of the curves, line bundles, and isospectral tori (Jacobians) when those are embedded as concrete objects in a projective space.

There are several reasons for studying geometric realizations of the finite-gap operator theory. The classical theory of curves and Jacobians is very beautiful, and an interesting statement about abstract curves and line bundles should be worth repeating about concrete representations. Furthermore, when one integrable system, like the stationary Lax equation $[L, B] = 0$, is

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THE SHAPE OF STATIONARY DISLOCATIONS

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Received 21 October 1985; accepted for publication 25 October 1985

It is shown that the structure of the stationary dislocations which occur in a wide variety of patterns in nonequilibrium systems is given by a self-similar solution of the universal Cross–Newell equation.

There has been a tremendous resurgence of interest lately in the structure and properties of symmetry breaking instabilities in driven systems far from equilibrium. The planforms and patterns which emerge from these instabilities are observed in a wide variety of physical situations, from convection in fluids, in liquid crystals and in binary mixtures to the solidification processes in undercooled liquids. Of particular interest are patterns which form in large aspect ratio systems; for example, the Rayleigh–Bénard convection of a Boussinesq fluid in a box wide enough to contain many rolls. Although for certain parameter ranges and in infinite horizontal geometries, there exist stable, fully nonlinear, spatially periodic solutions of the Oberbeck–Boussinesq equations

$$\boldsymbol{u}(\boldsymbol{x}, t) = f(\theta, A, R) , \qquad (1)$$

which correspond to a field of infinitely long, straight, parallel rolls, the patterns which are typically seen are much more complicated, involving patches of curved rolls, defects such as roll dislocations and grain boundaries. In an effort to treat the statics and slow dynamics of these patterns, Cross and Newell [1] developed a theory which averages over the detailed local structure of the rolls and concentrates on the global and universal properties of the pattern itself. The starting point of the theory is the existence and stability of the solution (1) representing the underlying roll structure. Here f is 2π periodic in the phase θ whose gradient is the constant wavevector $\boldsymbol{k}$. The periodicity demand gives rise to a relation (equivalent to the frequency dependency on amplitude in nonlinear oscillators)

$$\sigma(k, A, R) = 0 \qquad (2)$$

between the wavenumber $k = |\boldsymbol{k}|$, the amplitude A and the stress parameter R which in the case of convection is the Rayleigh number. The appearance of the modulus of $\boldsymbol{k}$ reflects the rotational symmetry. The regions of stability of these solutions in the R, k plane have been mapped out by Busse [2] (the Busse balloon).

The Cross–Newell theory develops a universal phase equation for the slowly changing wavevector $\boldsymbol{k}$, a change necessitated by the influence of distant, but finitely distant, horizontal boundaries to which the roll axis is perpendicular (if the thermal contact between wall and fluid is good). For order one values of $R - R_c$ (R_c is the transition value at which rolls first appear), the amplitude is still determined algebraically in terms of the phase gradient by (2). In the absence of the mean drift effect, which we do not consider here, the phase equation is

$$\tau(k)\theta_t + \nabla(\boldsymbol{k}B(k)) + F(k)(D_1 D_2 + D_2 D_1)A + ... = 0 , \qquad (3)$$

where the functions $\tau(k) > 0$, $B(k)$ and $F(k)$ are calculated easily from the particular model of interest (see ref. [1]) and the operators D_1 and D_2 are $2\boldsymbol{k}\cdot\nabla + \nabla\cdot\boldsymbol{k}$ and ∇^2, respectively. The function $B(k)$ always

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CHAPTER 4

AN APPLICATION: THE ROLL-UP OF A FINITE VORTEX LAYER

The evolution and regularity of vorticity distributions with varying degrees of smoothness are of great interest to those who study the Euler equations of fluid flow. A periodically perturbed vortex sheet, for example, is now widely believed to acquire a curvature singularity in finite time. Using the methods developed in Chapters 2 and 3 for the highly accurate application of boundary integral methods to multiply connected domains, the regularity of a thin, periodic layer of constant vorticity is investigated numerically. Numerical results suggest that, like the vortex sheet, the interfaces develop a curvature singularity, but now only in infinite time.

1. Background

The simplest model of a high Reynolds number shear layer is a surface of discontinuity, or vortex sheet, between two shearing, inviscid, irrotational fluids as pictured schematically in Figure 11. The instability of such an interface is prototypic and is the well-known Kelvin-Helmholtz instability. For simplicity, assume the fluid is of uniform density, and that $\bar{u} \to \pm U\hat{j}$ as $y \to \pm\infty$. Then the linear evolution of a normal mode of amplitude $A(k,t)$ and wave number k is given by

ORDER PRESERVING APPROXIMATIONS TO SUCCESSIVE DERIVATIVES OF PERIODIC FUNCTIONS BY ITERATED SPLINES

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CHAOS AND COHERENT STRUCTURES IN PARTIAL DIFFERENTIAL EQUATIONS

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This paper addresses the possible connections between chaos, the unpredictable behavior of solutions of finite dimensional systems of ordinary differential and difference equations and turbulence, the unpredictable behavior of solutions of partial differential equations. It is dedicated to Martin Kruskal on the occasion of his 60th birthday.

1. Introduction

The chaos that occurs in p.d.e.'s
cannot be fathomed by legalese
so we apply Occam's razor
and using a laser
study structures in ring cavities

An appealing idea of modern dynamics is that the complicated and apparently stochastic time behavior of large and even infinite-dimensional nonlinear systems is in fact a manifestation of a deterministic flow on a low-dimensional chaotic attractor. If the system is indeed low dimensional, it is natural to ask whether one can identify the physical characteristics such as the spatial structure of those few active modes which dominate the dynamics. Our thesis is that these modes are closely related to and best described in terms of asymptotically robust, multiparameter solutions of the nonlinear governing equations. We find it hard to define this robust nature precisely, but loosely speaking the idea is that these solutions are very stable and resilient asymptotic states. They may be coherent lumps like solitons and solitary waves. They may have the form of coherent wave packets. They may have self-similar form. They need not necessarily be the asymptotic states which develop as t tends to infinity; structures which develop singularities in finite time like those involved in the collapse of Langmuir waves or in filamentation in nonlinear optics are also candidates. For example, singular solutions of the Euler equations may be useful in understanding the behavior of the Navier–Stokes equations at high Reynolds numbers. Singular solutions like defects and dislocations certainly do play important roles in the pattern formations arising in continuum and condensed matter physics. The key idea is that each of these structures is a natural asymptotic state that, by virtue of the various force balances in the governing equations, develops an identity which does not easily decay or disperse away.

One can envision two types of chaos occurring. The first is a *phase* or *weak* turbulence which arises when there is an endless competition between equally resilient, localized coherent structures which are infinite time asymptotic states and which are initiated at random at various parts of the physical domain. Examples of this type of turbulence are solitary waves in the one-dimensional complex envelope equation, Rayleigh–Bénard roll patterns with different orientations and the oscillatory skew varicose states in low Prandtl number convection. It is to be expected that such dynamics may be low dimensional. The second type of chaos is much more dramatic and, for want of a better word, may be described as an

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